# EE-472 Smart Grids Technologies
# Module 2 Quiz (Graded)
# 22. 04. 2024

With Solutions

Student Name: ______________________

Sciper Number: ☐☐☐☐☐☐

**Question 1**

Not yet answered

Marked out of 10.00

Consider a single-phase electrical network of $N$ buses and $L$ lines. The admittance matrix $\bar{\mathbf{Y}}$ can expressed in terms of the incidence matrix $\mathbf{A}_{\mathfrak{B}}$ and the primitive branch and shunt admittance matrices, $\bar{\mathbf{Y}}_{\mathcal{L}}$ and $\bar{\mathbf{Y}}_{\mathcal{T}}$ respectively, as:

$$\bar{\mathbf{Y}} = \mathbf{A}_{\mathfrak{B}}^{\top}\bar{\mathbf{Y}}_{\mathcal{L}}\mathbf{A}_{\mathfrak{B}} + \bar{\mathbf{Y}}_{\mathcal{T}}.$$

Select the **incorrect** statement.

- ○ a. The diagonal elements of the primitive shunt admittance matrix are equal to the sum of the admittances of the lumped shunt elements connecting a node (e.g., $n$) and the ground $g$.
- ☒ b. The diagonal elements of the primitive branch admittance matrix are equal to the negative sum of the admittances of the lumped branch elements connecting two different nodes (e.g., $m$ and $n$).
- ○ c. The diagonal $(n, n)-$th element of the admittance matrix is the sum of all branch and shunt admittances connected to the respective node $n$.
- ○ d. The incidence matrix has dimensions $L \times N$.

**Question 2**

Not yet answered

Marked out of 10.00

Let $\bar{Z}'_{sc}$ and $\bar{Z}''_{sc}$ denote short-circuit impedances in $\Omega$ referred to the primary and secondary side, respectively, of a transformer with ratio $n \neq 1$.

In per-unit, corresponding values $\bar{z}'_{sc}$ and $\bar{z}''_{sc}$ are obtained using the base impedances $Z_{b,1}$ and $Z_{b,2}$ derived using an arbitrary base power $A_b$ and base voltages equal to the nominal voltages of the transformer's primary and secondary side, i.e., $V_{b,1} = V_{n,1}$ and $V_{b,2} = V_{n,2}$.

Consider the following statements:

1. $\bar{Z}'_{sc}$ and $\bar{Z}''_{sc}$ are equal.
2. $\bar{z}'_{sc}$ and $\bar{z}''_{sc}$ are equal.

Say which statement is correct.

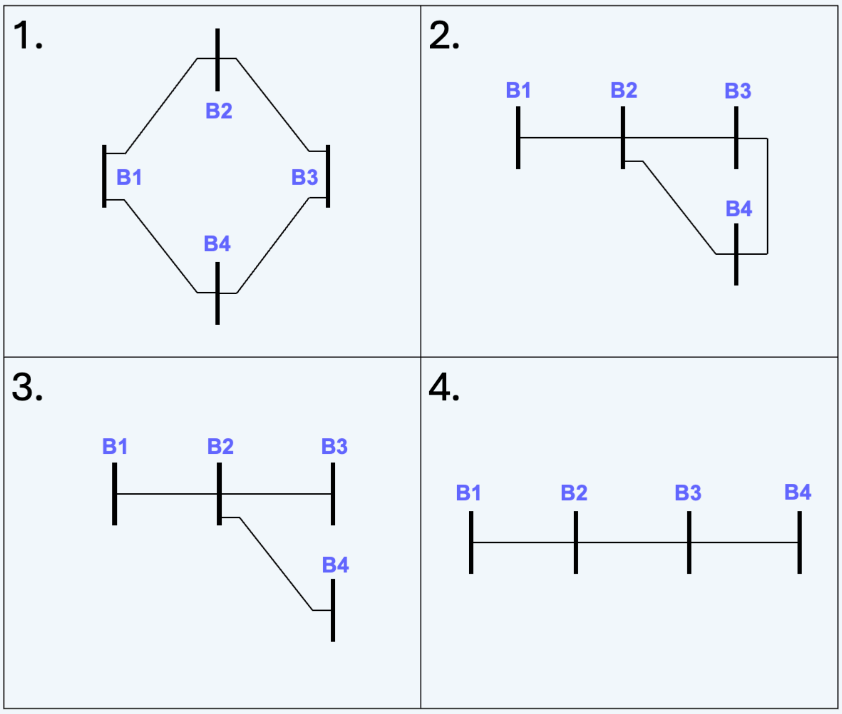
- ☒ a. Only 2.
- ○ b. Both 1 and 2.
- ○ c. Only 1.
- ○ d. None.

**Question 3**

Not yet answered

Marked out of 10.00

Among four given admittance matrices $\bar{\mathbf{Y}}^a, \bar{\mathbf{Y}}^b, \bar{\mathbf{Y}}^c$ and $\bar{\mathbf{Y}}^d$ where non-zero elements are indicated as $\bar{Y}_{ij}^x$, find the correct matching with four different topologies of four-bus networks.




$$\bar{\mathbf{Y}}^a = \begin{bmatrix} \bar{Y}_{11}^a & \bar{Y}_{12}^a & 0 & 0 \\ \bar{Y}_{21}^a & \bar{Y}_{22}^a & \bar{Y}_{23}^a & 0 \\ 0 & \bar{Y}_{32}^a & \bar{Y}_{33}^a & \bar{Y}_{34}^a \\ 0 & 0 & \bar{Y}_{43}^a & \bar{Y}_{44}^a \end{bmatrix}$$

$$\bar{\mathbf{Y}}^b = \begin{bmatrix} \bar{Y}_{11}^b & \bar{Y}_{12}^b & 0 & 0 \\ \bar{Y}_{21}^b & \bar{Y}_{22}^b & \bar{Y}_{23}^b & \bar{Y}_{24}^b \\ 0 & \bar{Y}_{32}^b & \bar{Y}_{33}^b & 0 \\ 0 & \bar{Y}_{42}^b & 0 & \bar{Y}_{44}^b \end{bmatrix}$$

$$\bar{\mathbf{Y}}^c = \begin{bmatrix} \bar{Y}_{11}^c & \bar{Y}_{12}^c & 0 & 0 \\ \bar{Y}_{21}^c & \bar{Y}_{22}^c & \bar{Y}_{23}^c & \bar{Y}_{24}^c \\ 0 & \bar{Y}_{32}^c & \bar{Y}_{33}^c & \bar{Y}_{34}^c \\ 0 & \bar{Y}_{42}^c & \bar{Y}_{43}^c & \bar{Y}_{44}^c \end{bmatrix}$$

$$\bar{\mathbf{Y}}^d = \begin{bmatrix} \bar{Y}_{11}^d & \bar{Y}_{12}^d & 0 & \bar{Y}_{14}^d \\ \bar{Y}_{21}^d & \bar{Y}_{22}^d & \bar{Y}_{23}^d & 0 \\ 0 & \bar{Y}_{32}^d & \bar{Y}_{33}^d & \bar{Y}_{34}^d \\ \bar{Y}_{41}^d & 0 & \bar{Y}_{43}^d & \bar{Y}_{44}^d \end{bmatrix}$$

*Notation: $(k - m)$ means that grid topology $k \in \{1, 2, 3, 4\}$ can be described by the admittance matrix with superscript $m \in \{a, b, c, d\}$. E.g., answer (1-a) states that grid topology 1 would correspond to admittance matrix $\bar{\mathbf{Y}}^a$.*

- a. (1-c), (2-a), (3-d), (4-b)
- b. (1-a), (2-b), (3-d), (4-c)
- c. (1-d), (2-c), (3-b), (4-a)
- d. None of the above.

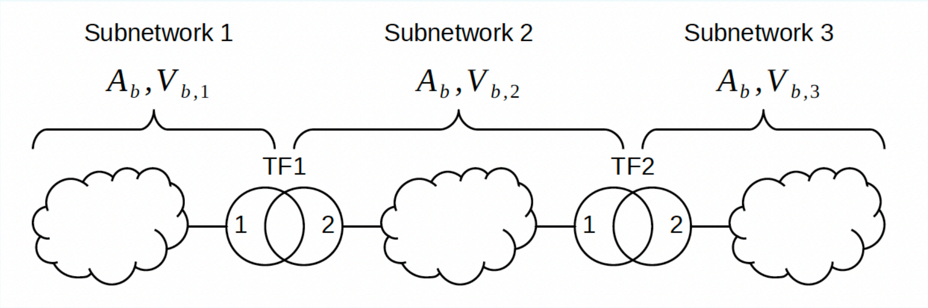
Question **4**

Not yet answered

Marked out of 10.00

Two transformers, TF1 and TF2, with nominal voltage ratios $V_{n,1}^{TF1}/V_{n,2}^{TF1}$ and $V_{n,1}^{TF2}/V_{n,2}^{TF2}$, respectively, divide a network into three subnetworks at different voltage levels (see Figure below). As in the lab on admittance matrix calculus, transformers are modelled, in absolute units, by short-circuit impedance and ideal transformer with corresponding nominal ratio. We neglect the zero-load admittance of transformers.

A base voltage triple $(V_{b,1}, V_{b,2}, V_{b,3})$ is called *model-friendly* if both transformers' per-unit equivalent circuits contain only branch elements, i.e., per-unit short-circuit impedance.




Consider the following statements:

1. $(V_{b,1}^{\star}, V_{b,2}^{\star}, V_{b,3}^{\star})$ is model-friendly if and only if, for any $k > 0$, $(kV_{b,1}^{\star}, kV_{b,2}^{\star}, kV_{b,3}^{\star})$ is model-friendly.
2. A model-friendly base voltage triple exists only if $V_{n,2}^{TF1} = V_{n,1}^{TF2}$.

Which of the statements is correct?

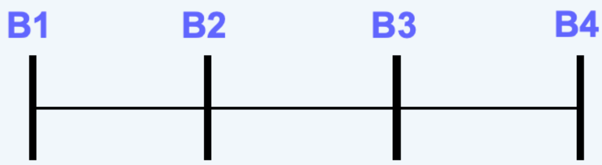
- ○ a. None.
- ○ b. Both 1 and 2.
- ○ c. Only 2.
- ☒ d. Only 1.

Question **5**

Not yet answered

Marked out of 10.00

A small radial network (shown in Figure below) where all the shunts are neglected is described by the admittance matrix $\bar{Y}$.




Knowing only the diagonal elements (in per-unit) $\bar{Y}_{11} = 1 - j1$ and $\bar{Y}_{22} = 2 - j3$, what is the value of $\bar{Y}_{23}$?

- ○ a. $-2 + j3$
- ○ b. It cannot be uniquely determined. We need additional matrix elements.
- ○ c. $-1 + j3$
- ☒ d. $-1 + j2$

**Question 6**

Not yet answered

Marked out of 10.00

Consider a small electrical grid of six buses. We assume that the system is balanced and consider a single-phase representation of the grid. For each bus, the imposed and unknown parameters are specified in the following table:

| Bus No. | Imposed Parameter 1 | Imposed Parameter 2 | Unknown Parameter 1 | Unknown Parameter 2 |
|---|---|---|---|---|
| 1 | $P_1$ | $Q_1$ | $\arg(\bar{V}_1)$ | $\lvert\bar{V}_1\rvert$ |
| 2 | $P_2$ | $Q_2$ | $\arg(\bar{V}_2)$ | $\lvert\bar{V}_2\rvert$ |
| 3 | $P_3$ | $\lvert\bar{V}_3\rvert$ | $\arg(\bar{V}_3)$ | $Q_3$ |
| 4 | $P_4$ | $Q_4$ | $\arg(\bar{V}_4)$ | $\lvert\bar{V}_4\rvert$ |
| 5 | $\arg(\bar{V}_5)$ | $\lvert\bar{V}_5\rvert$ | $P_5$ | $Q_5$ |
| 6 | $P_6$ | $\lvert\bar{V}_6\rvert$ | $\arg(\bar{V}_6)$ | $Q_6$ |

To solve the Load-flow equations, one can write the system equations in both polar and Cartesian coordinates and use the Newton-Raphson algorithm. For both formulations, in the iterative process, we compute the reduced Jacobian matrix that corresponds to load-flow equations excluding the identities.

Consider the following statements:

1. In Cartesian coordinates, the reduced Jacobian matrix has dimensions $10 \times 10$.
2. In polar coordinates, the reduced Jacobian matrix has dimensions $8 \times 8$.

Which statement is correct?

- ☒ a. Both 1 and 2.
- ○ b. Only 1.
- ○ c. Only 2.
- ○ d. None.

**Question 7**

Not yet answered

Marked out of 10.00

Which of the following statements about Load-Flow approximations is wrong?

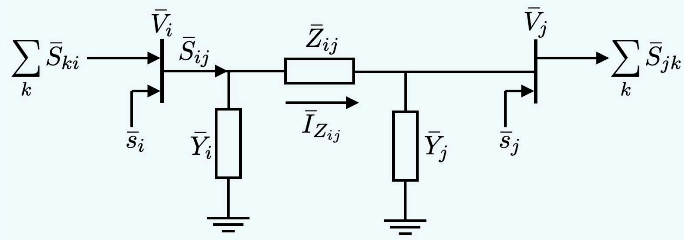
- ○ a. Solving the load-flow by applying the Stott approximation does not require updating the Jacobian matrix in each iteration.
- ☒ b. The Carpentier approximation is suitable for both transmission and distribution systems since the injected active powers depend mainly on the phases of the voltages and the injected reactive powers depend mainly on the modules of the voltages.
- ○ c. In transmission systems, the system operator can use the DC load-flow approximation solution (i.e., the voltage angles $\theta$) for initialization of the Newton-Rhapson algorithm for non-approximated load-flow.
- ○ d. If the linearized load-flow problem has a solution, then one can uniquely identify all voltage sensitivity coefficients, $K_P^{il} = \frac{\partial|\bar{V}_i|}{\partial P_l}$ and $K_Q^{il} = \frac{\partial|\bar{V}_i|}{\partial Q_l}$.

**Question 8**

Not yet answered

Marked out of 10.00

For the branch connecting nodes $i$ and $j$ in a generic network, the branch-flow equations are given by (★).



Branch connecting nodes $i$ and $j$.

$$\left.\begin{aligned}
\bar{S}_{ij} - \bar{Z}_{ij} i_{Z_{ij}} - \bar{Y}_i v_i - \bar{Y}_j v_j + \bar{s}_j &= \sum_k \bar{S}_{jk} \\
v_j = v_i + |\bar{Z}_{ij}|^2 i_{Z_{ij}} &- 2\Re\left[\underline{Z}_{ij}(\bar{S}_{ij} - \bar{Y}_i v_i)\right] \\
\sum_k \bar{S}_{ki} + \bar{s}_i &= \bar{S}_{ij} \\
i_{Z_{ij}} &= \frac{\left|\bar{S}_{ij} - \bar{Y}_i v_i\right|^2}{v_i} \\
v_i = \left|\bar{V}_i\right|^2, v_j = \left|\bar{V}_j\right|^2 &\text{ and } i_{Z_{ij}} = \left|\bar{I}_{Z_{ij}}\right|^2 \\
\arg\left(\bar{V}_i\right) - \arg\left(\underline{V}_j\right) &= \arg\left(v_i - \underline{Z}_{ij}(\bar{S}_{ij} - \bar{Y}_i v_i)^2\right)
\end{aligned}\right\} (\bigstar)$$

Which of the following statements is correct?

- ○ a. A disadvantage of the branch-flow model is that we cannot retrieve the voltage angles, but only the voltage and current magnitudes.
- ○ b. The system of equations (★) can be written only for $\pi$-equivalent circuits of lines but not for $\pi$-equivalent circuits of transformers (modelled in per-unit).
- ☒ c. The branch flow model can be used for both meshed and radial (i.e., without loops) networks.
- ○ d. The power balance over a branch connecting two nodes has a linear relationship over the complex nodal voltages and the complex branch current.

**Question 9**

Not yet answered

Marked out of 10.00

We consider two identical electrical distribution systems with the same placement of the PMUs where:

- In system 1, all of the measurements have a Gaussian-type probability density function (p.d.f).
- In system 2, one PMU produces measurements in which the measurement noise does **not** follow a Gaussian distribution.

All PMU measurements are used in the maximum likelihood estimation (MLE) method to obtain the most likely system state.

As seen in the lecture, the Log-Likelihood Function $L$ is given by:

$$L = \log f_m(\mathbf{z}) = \sum_{i=1}^{m} \log f(z_i) = -\frac{1}{2} \sum_{i=1}^{m} \left(\frac{z_i - \mu_i}{\sigma_i}\right)^2 - \frac{m}{2} \log 2\pi - \sum_{i=1}^{m} \log \sigma_i$$

Therefore, the MLE will maximize the function $L$ for a given set of measurements $z_1, z_2, \ldots, z_m$:

$$\underset{x}{\text{maximize}} \ \log f_m(\mathbf{z}) \quad \text{OR} \quad \underset{x}{\text{minimize}} \sum_{i=1}^{m} \left(\frac{z_i - \mu_i}{\sigma_i}\right)^2$$

Which of the following statements is correct?

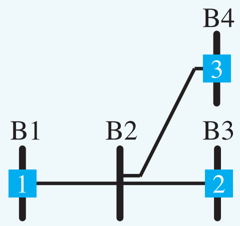
- ○ a. The distribution of the measurement noise has no influence on the performance of the MLE method.
- ○ b. As long as the measurement noise is zero-biased, i.e. the mean $\mu$ is zero, the MLE method will have the same result for both systems.
- ☒ c. The MLE method cannot guarantee that the estimated states of system 2 are the maximum likelihood estimates.
- ○ d. The MLE method cannot infer the state of system 2.

**Question 10**

Not yet answered

Marked out of 10.00

Let's consider the following 4-node grid that is equipped with 3 PMUs.




Assume the measurement matrix is composed as: $\mathbf{H} = \begin{bmatrix} \mathbf{H}_V \\ \mathbf{H}_I \end{bmatrix}$.

The state vector is constructed as: $\mathbf{x} = \begin{bmatrix} \Re\{\overline{\mathbf{V}}\} \\ \Im\{\overline{\mathbf{V}}\} \end{bmatrix}$ and the measurement vector as: $\mathbf{z} = \begin{bmatrix} \mathbf{z}_V \\ \mathbf{z}_I \end{bmatrix}$, with $\mathbf{z}_V = \begin{bmatrix} \Re\{\widetilde{\mathbf{V}}\} \\ \Im\{\widetilde{\mathbf{V}}\} \end{bmatrix}$ and $\mathbf{z}_I = \begin{bmatrix} \Re\{\widetilde{\mathbf{I}}\} \\ \Im\{\widetilde{\mathbf{I}}\} \end{bmatrix}$.

Furthermore, the compound admittance matrix of the system is given as $\overline{\mathbf{Y}} = \mathbf{G} + j\mathbf{B}$, where the non-zero $(i, j)$-th element is expressed as $g_{ij} + jb_{ij}$.

Which of the following matrices correctly represents the measurement model of the above-given system?

$$\mathbf{H}^a = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ g_{12} & g_{11} & 0 & 0 & -b_{12} & -b_{11} & 0 & 0 \\ 0 & g_{23} & g_{22} & 0 & 0 & -b_{23} & -b_{22} & 0 \\ 0 & 0 & g_{34} & g_{33} & 0 & 0 & -b_{34} & -b_{33} \\ b_{12} & b_{11} & 0 & 0 & g_{12} & g_{11} & 0 & 0 \\ 0 & b_{23} & b_{22} & 0 & 0 & g_{23} & g_{22} & 0 \\ 0 & 0 & b_{34} & b_{33} & 0 & 0 & g_{34} & g_{33} \end{bmatrix}, \quad \mathbf{H}^b = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ g_{11} & g_{12} & 0 & 0 & -b_{11} & -b_{12} & 0 & 0 \\ 0 & g_{22} & g_{23} & 0 & 0 & -b_{22} & -b_{23} & 0 \\ 0 & 0 & g_{33} & g_{34} & 0 & 0 & -b_{33} & -b_{34} \\ b_{11} & b_{12} & 0 & 0 & g_{11} & g_{12} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 & 0 & g_{22} & g_{23} & 0 \\ 0 & 0 & b_{33} & b_{34} & 0 & 0 & g_{33} & g_{34} \end{bmatrix},$$

$$\mathbf{H}^c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ g_{11} & g_{12} & 0 & 0 & b_{11} & b_{12} & 0 & 0 \\ 0 & g_{22} & g_{23} & 0 & 0 & b_{22} & b_{23} & 0 \\ 0 & 0 & g_{33} & g_{34} & 0 & 0 & b_{33} & b_{34} \\ -b_{11} & -b_{12} & 0 & 0 & g_{11} & g_{12} & 0 & 0 \\ 0 & -b_{22} & -b_{23} & 0 & 0 & g_{22} & g_{23} & 0 \\ 0 & 0 & -b_{33} & -b_{34} & 0 & 0 & g_{33} & g_{34} \end{bmatrix}$$

- [x] a. None of the above.
- ○ b. $\mathbf{H}^c$
- ○ c. $\mathbf{H}^a$
- ○ d. $\mathbf{H}^b$